Image Compression Using Discrete Cosine Transform & Discrete Wavelet Transform

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Abstract— Image compression plays a vital role in digital image processing. The need for image compression becomes apparent when number of bits per image are computed resulting from typical sampling rates and. quantization methods. For example, the amount of storage required for given images is (i) a low resolution, TV quality, color video image which has 512 x 512 pixels/color,8 bits/pixel, and 3 colors approximately consists of 6 x 10^6 bits;(ii) a 24 x 36 mm negative photograph scanned at 12 x 10^{-6}mm:3000 x 2000 pixels/color, 8 bits/pixel, and 3 colors nearly contains 144 x 10^6 bits; (3) a 14 x 17 inch radiograph scanned at 70 x 10^{-6}mm: 5000 x 6000 pixels, 12 bits/pixel nearly contains 360 x 10^6 bits. Thus storage of even a few images could cause a problem.DCT is one of the transforms used for lossy image compression. This paper reveals a study of the mathematical equations of the DCT and its uses with image compression.

Index Terms—CR, DCT, IDCT, ISO, JPEG, TV

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1 INTRODUCTION

Image compression is very important for efficient transmission and storage of images . Demand for communication of multimedia data through the telecommunications network and accessing the multimedia data through Internet is growing explosively[14].With the use of digital cameras, requirements for storage, manipulation, and transfer of digital images, has grown explosively . These image files can be very large and can occupy a lot of memory. A gray scale image that is 256 x 256 pixels has 65, 536 elements to store, and a a typical 640 x 480 color image has nearly a million. Downloading of these files from internet can be very time consuming task. Image data comprise of a significant portion of the multimedia data and they occupy the major portion of the communication bandwidth for multimedia communication. Therefore development of efficient techniques for image compression has become quite necessary [9]. Fortunately, there are several methods of image compression available today. These fall into two general categories: lossless and lossy image compression. JPEG process is a widely used form of lossy image compession that centers around the Discrete Cosine Transform. The DCT works by separating images into parts of differing frequencies. During a step called quantization, where part of compression actually occurs, the less important frequencies are discarded, hence the use of the term “lossy”. Then only the
most important frequencies that remain are used to retrieve the image in the decompression process. As a result, reconstructed images contain some distortion, but these levels can be adjusted during the compression stage.

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2. Process

The following is a general overview of the JPEG process. Later, we will take the reader through a detailed tour of JPEG’s method so that a more comprehensive understanding of the process may be acquired.

1. The image is broken into 8*8 blocks of pixels.
2. Working from left to right, top to bottom, the DCT is applied to each block.
3. Each block is compressed through quantization.
4. The array of compressed blocks that constitute the image is stored in a drastically reduced amount of space.
5. When desired, the image is reconstructed through decompression, a process that uses the Inverse Discrete Cosine Transform(IDCT)

The DCT Equation

The DCT (1) computes the i,jth entry of the DCT of an image.

3. Image Compression using Discrete Cosine Transform

JPEG stands for the Joint Photographic Experts Group, a standards committee that had its origins within the International Standard Organization (ISO).JPEG provides a compression method that is capable of compressing continuous-tone image data with a pixel depth of 6 to 24 bits with reasonable speed and efficiency.JPEG may be adjusted to produce very small, compressed images that are of relatively poor quality in appearance but still suitable for many applications. Conversely, JPEG is capable of producing very high-quality compressed images that are still far smaller than the original uncompressed data.

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JPEG is primarily a lossy method of compression.JPEG was designed specifically to discard information that the human eye cannot easily see. Slight changes in color are not perceived well by the human eye, while slight changes in intensity (light and dark) are. Therefore JPEG's lossy encoding tends to be more frugal with the gray-scale part of an image and to be more frivolous with the color[21].DCT separates images into parts of different frequencies where less important frequencies are discarded through quantization and important frequencies are used to retrieve the image during decompression. Compared to other input dependent transforms, DCT has many advantages: (1) It has been implemented in single integrated circuit; (2) It has the ability to pack most information in fewest coefficients; (3) It minimizes the block like appearance called blocking artifact that results when boundaries between sub-images become visible [11].

The forward 2D_DCT transformation is given by (1):

$$C(u,v)=D(u)D(v)\sum_{\substack{k\\x=0}}^{N-1}\sum_{\substack{k\\y=0}}^{N-1} f(x,y)\cos[(2x+1)u\pi/2N]\cos[(2y+1)v\pi/2N] \quad (1)$$

where u,v=0,1,2,3,---N-1

The inverse 2D-DCT transformation is given by the following equation

$$f(x,y)=\sum_{\substack{k\\u=0}}^{N-1}\sum_{\substack{k\\v=0}}^{N-1} D(u)D(v)\, f(x,y)\cos[(2x+1)u\pi/2N]\cos[(2y+1)v\pi/2N] \quad (2)$$

where

D(u)=(1/N) ^1/2 for u=0

D(v)=2(/N)^1/2 for u=1,2,3…….,(N-1)

3.1 JPEG Process

- Original image is divided into blocks of 8 x 8.
- Pixel values of a black and white image range from 0-255 but DCT is designed to work on pixel values ranging from -128 to 127 .Therefore each block is modified to work in the range.
- is used to calculate DCT matrix.
- DCT is applied to each block by multiplying the modified block with DCT matrix on the left and transpose of DCT matrix on its right.
- Each block is then compressed through quantization.
- Quantized matrix is then entropy encoded.
- Compressed image is reconstructed through reverse process.
- Inverse DCT is used for decompression[11].

3.2 Quantization:

Quantization is achieved by compressing a range of values to a single quantum value. When the number of discrete symbols in a given stream is reduced, the stream becomes more compressible. A quantization matrix is used in combination with a DCT coefficient matrix to carry out transformation. Quantization is the step where most of the compression takes place.DCT really does not compress the image because it is almost lossless. Quantization makes use of the fact that higher frequency components are less important than low frequency components. It allows varying levels of image compression and quality through selection of specific quantization matrices. Thus quality levels ranging from 1 to 100 can be selected, where 1 gives the poorest image quality and highest compression, while 100 gives the best quality and lowest compression. As a result quality to compression ratio can be selected to meet different needs.JPEG committee suggests matrix with quality level 50 as standard matrix. For obtaining quantization matrices with other quality levels, scalar multiplications of standard quantization matrix are used. Quantization is achieved by dividing transformed image matrix by the quantization matrix used. Values of the resultant matrix are then rounded off. In the resultant matrix coefficients situated near the upper left corner have lower frequencies .Human eye is more sensitive to lower frequencies .Higher frequencies are discarded. Lower frequencies are used to reconstruct the image[11].

3.3 Entropy Encoding

After quantization, most of the high frequency coefficients are zeros. To exploit the number of zeros, a zig-zag scan of the matrix is used yielding to long string of zeros. Once a block has been converted to a spectrum and quantized, the JPEG compression algorithm then takes the result and converts it into a one dimensional linear array, or vector of 64 values, performing a zig-zag scan .

3.4Huffman coding:

The basic idea in Huffman coding is to assign short codewords to those input blocks with high probabilities and long codewords to those with low probabilities. A Huffman code is designed by merging together the two least probable characters, and repeating this process until there is only one character remaining. A code tree is thus generated and the Huffman code is obtained from the labeling of the codetree.

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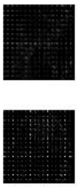
fig(3.1)

Original image

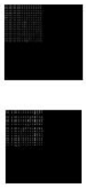


fig(3.2)

After 8 x 8 DCT

fig(3.3)

Original image



fig(3.4)

After 4 x 4 DCT

fig(3.5)

Original Lena image

fig(3.6)

Compressed Lena image with 4 coefficients

fig(3.7)

Compressed image with 16 coefficients

fig(3.8)

Compressed image with 25 coefficients

fig(3.9)

Compressed image with 40 coefficients

fig(3.10)

Compressed image with 50 coefficients

3.5 Results and Discussions:

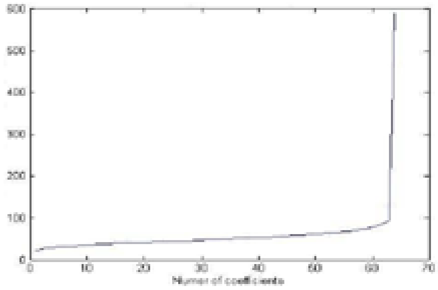
Results obtained after performing DCT of various orders on original images are shown. Fig(3.1) shows original lena image. Images obtained after applying 8 x 8 DCT are as shown in Fig(3.2) whereas Fig(3.4) shows image obtained for same original image after applying 4 x 4 DCT. .Fig(3.6) to Fig(3.10) show compressed images for the original Lena image after taking various number of coefficients for quantization.As the number of coefficients increases quality of the image decreases whereas compression ratio continues to increase.Fig(3.11) shows that SNR value increases with number of coefficients

4.Image Compression using Discrete Wavelet Transform

Wavelet Transform has become an important method for image compression. Wavelet based coding provides sub

stantial improvement in picture quality at high compression ratios mainly due to better energy compaction property of wavelet transforms. Wavelet transform partitions

a signal into a set of functions called wavelets. Wavelets are obtained from a single prototype wavelet called mother wavelet by dilations and shifting. The wavelet transform is computed separately for different segments of the time-domain signal at different frequencies.



fig(3.11)

SNR vs. No. of coefficients

4.1 Subband coding:

A signal is passed through a series of filters to calculate DWT. Procedure starts by passing this signal sequence through a half band digital low pass filter with impulse response h(n).Filtering of a signal is numerically equal to convolution of the tile signal with impulse response of the filter.

$$x[n]*h[n]= \sum_{k=-\infty}^{\infty} x[k].h[n-k] \quad (3)$$

A half band low pass filter removes all frequencies that are above half of the highest frequency in the tile signal. Then the signal is passed through high pass filter.The two filters are related to each other as

$h[L-1-n]=(-1)^n g(n)$

Filters satisfying this condition are known as quadrature mirror filters. After filtering half of the samples can be eliminated since the signal now has the highest frequency as half of the original frequency. The signal can therefore be subsampled by 2,simply by discarding every other sample. This constitutes 1 level of decomposition and can mathematically be expressed as in (4) & (5)

$$Y1[n]= x[k]h[2n-k]\ k \quad (4)$$
$$Y2[n]= x[k]g[2n+1-k]\ k \quad (5)$$

where y1[n] and y2[n] are the outputs of low pass and high pass filters, respectively after subsampling by 2.

This decomposition halves the time resolution since only half the number of sample now characterizes the whole signal. Frequency resolution has doubled because each output has half the frequency band of the input. This process is called as sub band coding. It can be repeated further to increase the frequency resolution as shown by the filter bank.

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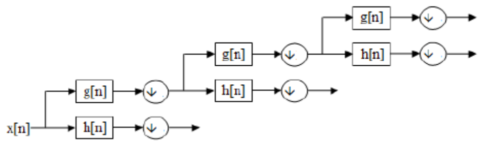

Fig(4.1)

Filter Bank

4.2 Compression steps:

1. Digitize the source image into a signal s, which is a string of numbers.
2. Decompose the signal into a sequence of wavelet coefficients w.
3. Use threshold to modify the wavelet coefficients from w to w′.
4. Use quantization to convert w′ to a sequence q.
5. Entropy encoding is applied to convert q into a sequence e.

4.3Digitization

The image is digitized first. The digitized image can be characterized by its intensity levels, or scales of gray which range from 0(black) to 255(white), and its resolution, or how many pixels per square inch.

4.4Thresholding

In certain signals, many of the wavelet coefficients are close or equal to zero. Through threshold these coefficients are modified so that the sequence of wavelet coefficients contains long strings of zeros.

In hard threshold ,a threshold is selected. Any wavelet whose absolute value falls below the tolerance is set to zero with the goal to introduce many zeros without losing a great amount of detail.

4.5Quantization

Quantization converts a sequence of floating numbers w′ to a sequence of integers q.The simplest form is to round to the nearest integer.Another method is to multiply each number in w′ by a constant k, and then round to the nearest integer.Quantization is called lossy because it introduces error into the process, since the conversion of w′ to q is not one to one function.

Entropy encoding

With this method ,a integer sequence q is changed into a shorter sequence, with the numbers in e being 8 bit integers The conversion is made by an entropy encoding table. Strings of zeros are coded by numbers 1 through 100,105 and 106,while the non-zero integers in q are coded by 101 through 104 and 107 through 254.

4.6 DWT Results:

Results obtained below. Fig(4.3.1) shows original Lena image. Fig(4.3.2) to Fig(4.3.4) show compressed images for various threshold values. As threshold value increases blurring of image continues to increase.

Fig (4.3.1)

Original Lena image

Fig (4.3.2)

Compressed Image for threshold value 1

Fig 4.3.3)

Compressed Image for threshold value 2

Fig (4.3.4)

Compressed Image for threshold value 5

4. Results:

Comparisons of results for DCT and DWT based on various performance parameters :

Mean Squared Error (MSE) is defined as the square of differences in the pixel values between the corresponding pixels of the two images. Graph of Fig(4.4.1) shows that for DCT based image compression ,as the window size increases MSE increases proportionately whereas for DWT based image compression Fig(4.4.2) shows that MSE first decreases with increase in window size and then starts to increase slowly with finally attaining a constant value. Fig(4.4.3) and Fig(4.4.4) plot show required for compressing image with change in window size for DCT and DWT respectively. Fig(4.4.5) and Fig(4.4.6) indicate compression ratio with change in window size for DCT and DWT based image compression techniques respectively. Compression increases with increase in window size for DCT and decreases with increase in window size for DWT.

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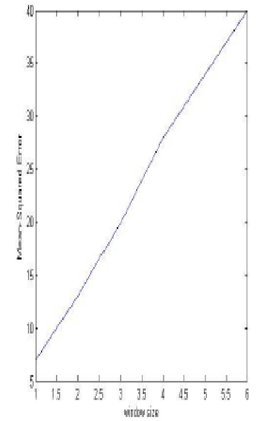
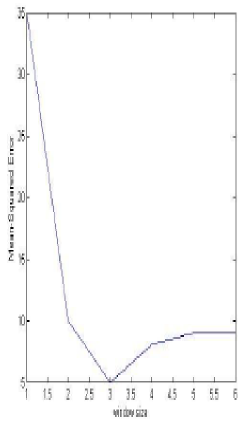


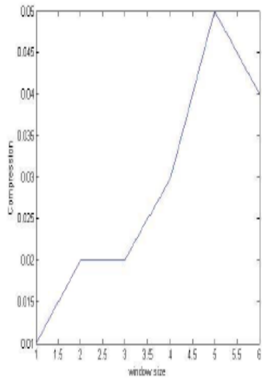
Fig(4.4.1)

Mean Squared Error vs. window size for DCT



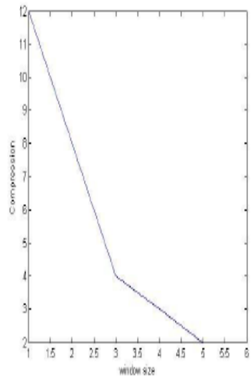
Fig(4.4.2)

Mean Squared Error vs. window size for DWT



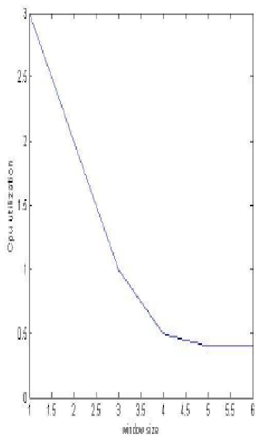
Fig(4.4.5)

Compression vs. window size for DCT



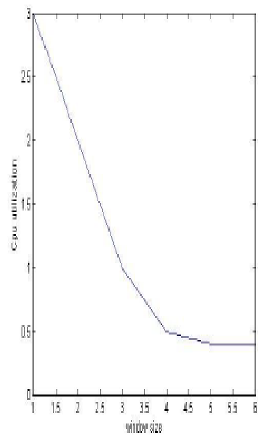
Fig(4.4.6)

Compression vs. window size for DWT



Fig(4.4.3)

Cpu Utilization vs window size for DCT



Fig(4.4.4)

Cpu Utilization vs window size for DWT

5. Conclusions:

In the thesis image compression techniques using DCT and DWT were implemented.

DCT is used for transformation in JPEG standard. DCT performs efficiently at medium bit rates. Disadvantage with DCT is that only spatial correlation of the pixels inside the single 2-D block is considered and the correlation from the pixels of the neighboring blocks is neglected. Blocks cannot be decorrelated at their boundaries using DCT. DWT is used as basis for transformation in JPEG 2000 standard. DWT provides high quality compression at low bit rates. The use of larger DWT basis functions or wavelet filters produces blurring near edges in images.

DWT performs better than DCT in the context that it avoids blocking artifacts which degrade reconstructed images. However DWT provides lower quality than JPEG at low compression rates.DWT requires longer compression time.

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